

PROPERTIES OF LOGARITHMS

Product Property

For any positive number m , n , and b where $b \neq 1$.

$$\log_b mn = \log_b m + \log_b n$$

Quotient Property

For any positive number m , n , and b where $b \neq 1$. $n \neq 0$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power Property

For any positive number m , n , and b where $b \neq 1$.

$$\log_b m^n = n \log_b m$$

→ single log

1. Condense: $\log_3 7 + \log_3 x$

$$\log_3 7x$$

2. Condense: $\log_5 4 + \log_5 3$

$$\log_5 4 \cdot 3 = \log_5 12$$

3. Expand: $\log_4 5x$

$$\log_4 5 + \log_4 x$$

multiple logs

4. Expand: $\log_6 36x$

$$\log_6 36 + \log_6 x$$

$$2 + \log_6 x$$

5. Condense: $\log_6 x - \log_6 y$

$$\log_6 \frac{x}{y}$$

6. Condense: $\log_2 45 - \log_2 9$

$$\log_2 \frac{45}{9} = \log_2 5$$

7. Expand: $\log_2 \frac{x}{y}$

$$\log_2 x - \log_2 y$$

8. Expand: $\log_5 \frac{125}{x}$

$$\log_5 125 - \log_5 x$$

9. Condense: $2 \log_8 x$

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$$\log_8 x^2$$

10. Condense: $\frac{1}{3} \log_5 x$

$$\log_5 x^{\frac{1}{3}} \rightarrow \log_5 \sqrt[3]{x}$$

11. Expand: $\log_5 x^8$

$$8 \log_5 x$$

12. Expand: $\log_3 9^7$

$$\log_3 9 = 2$$

$$7 \log_3 9 \rightarrow 7(2) \rightarrow \boxed{14}$$

Log Evaluating and Properties

Date _____

Period _____

Evaluate each expression.

1) $\log_5 25$

$$x=2$$

2) $\log_3 1$

$$3^x = 1$$
$$x=0$$

$$x=-3$$

3) $\log_{\frac{1}{5}} \frac{1}{25}$

$$\left(\frac{1}{5}\right)^x = \frac{1}{25} \quad x=2$$

4) $\log_6 \frac{1}{216}$

$$6^x = \frac{1}{216}$$

5) $\log_2 \frac{1}{16}$

$$2^x = \frac{1}{16} \quad x=-4$$

6) $\log_{49} \frac{1}{7}$

$$49^x = \frac{1}{7} \quad x=-\frac{1}{2}$$

bigger → smaller
+ neg.
Flip

7) $\log_{216} 36$

$$216^x = 36$$
$$6^{3x} = 6^2$$
$$3x=2$$
$$x=\frac{2}{3}$$

8) $\log_{125} 25$

$$125^x = 25$$
$$5^{3x} = 5^2$$
$$3x=2$$
$$x=\frac{2}{3}$$

Expand each logarithm.

9) $\log_2 (x \cdot y)$

$$\log_2 x + \log_2 y$$

10) $\ln 7^4$

$$4 \ln 7$$

$$\overbrace{\log 7^4}^4$$
$$4 \cdot \log 7$$

11) $\log_8 \frac{x}{y}$

$$\log_8 x - \log_8 y$$

12) $\log_5 (x \cdot y)$

$$\log_5 x + \log_5 y$$

13) $\log_2 a^2$
 $2 \log_2 a$

14) $\log_8 x^6$
 $6 \log_8 x$

15) $\log_2 \frac{u}{v}$
 $\log_2 u - \log_2 v$

16) $\log(a \cdot b)$
 $\log a + \log b$

17) $\log_3 \sqrt{12} \rightarrow \log_3 12^{1/2}$
 $\frac{1}{2} \log_3 12$

18) $\log_7 \sqrt[3]{5}$
 $\frac{1}{3} \log_7 5$ $\log_7 5^{1/3}$

Condense each expression to a single logarithm.

19) $\log_9 10 + \log_9 7$
 $\log_9 70$

20) $\log_4 7 - \log_4 6$
 $\log_4 \frac{7}{6}$

21) $4 \log_7 11$
 $\log_7 11^4$

22) $\log_7 2 + \log_7 3$
 $\log_7 6$

23) $\log_2 10 - \log_2 3$
 $\log_2 \frac{10}{3}$

24) $5 \log_9 a$
 $\log_9 a^5$

25) $5 \log_9 7$
 $\log_9 7^5$

26) $\log_4 6 - \log_4 5$
 $\log_4 \frac{6}{5}$

27) $\frac{\log_9 x}{2}$
 $\frac{1}{2} \cdot \log_9 x$
 $\log_9 x^{1/2}$ or $\log_9 \sqrt{x}$

28) $\frac{\log_3 8}{2}$
 $\log_3 8^{1/2}$ or $\log_3 \sqrt{8}$