Today was a shortened class due to EOC Scheduling. We quickly discussed the concepts of holes and Vertical Asymptotes in rational functions. We then practiced some problems. The homework is to complete the odd problems from the worksheet at the end of this file.
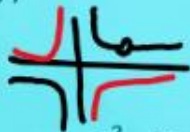
Graphing Rational Functions: Holes

Steps:
1. Factor completely & look for common factors to cancel.
2. If you can cancel a common factor, it creates a hole in the graph.
3. Set the cancelled factor =0 & solve. (This is the x-coordinate of the hole) ▬
4. Plug that x-value you just found into the remaining function & solve for y. (This is the y-coordinate of the hole.)
5. Write the hole as an ordered pair. (x, y)

Examples: Find the hole if possible.

1. $f(x) = \dfrac{(x+2)(x-3)}{(x+2)(x-5)}$

$f(x) = \dfrac{(x-3)}{(x-5)}$

$X+2=0$
$X=-2$

$Y = \dfrac{-2-3}{-2-5} = \dfrac{5}{7}$

$hole: \left(-2, \dfrac{5}{7}\right)$

2. $f(x) = \dfrac{x^2-4x-12}{x^2+3x+2} = \dfrac{(X-6)(X+2)}{(X+1)(X+2)}$

$X+2=0$
$X=-2$

$Y = \dfrac{(-2-6)}{(-2+1)} = \dfrac{-8}{-1} = 8$

$hole: (-2,8)$

3. $f(x) = \dfrac{x^2-4x}{x^2-5x+4} = \dfrac{x(x-4)}{(x-4)(x-1)}$

$X-4=0$
$X=4$

$Y = \dfrac{4}{4-1} = \dfrac{4}{3}$

$hole: \left(4, \dfrac{4}{3}\right)$

4. $f(x) = \dfrac{2x}{x^2+2x+1} = \dfrac{2x}{(x+1)(x+1)}$

No hole

Graphing Rational Functions: Vertical Asymptotes

Steps:

After you look for holes...
1. Set the denominator = 0.
2. Solve for x.

Write vertical asymptotes as x = #

Examples: Find the vertical asymptotes.

1. $f(x) = \frac{5}{x-3}$

$$x - 3 = 0$$
$$x = 3 \leftarrow V.A.$$

V.A.: $\underline{X = 3}$ must be an equation
needs equal sign

2. $f(x) = \frac{2x}{x^2 - 7x + 12}$

3. $f(x) = \frac{x^2 - x - 6}{x^2 + 3x + 2}$

4. $f(x) = \frac{2x}{x^2 + 2x} = \frac{2x}{x(x+2)} = \frac{2}{x+2}$

$$x + 2 = 0$$

VA: $\underline{x = -2}$

Name_____
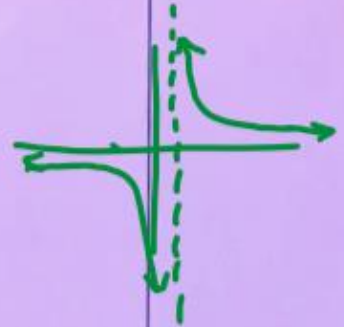
Finding Holes and Vertical Asymptotes

Date_____ Period____

For each function, identify the holes and vertical asymptote(s).

1) $f(x) = \dfrac{(x-4)(x+5)}{(x-4)(x-1)} = \dfrac{x+5}{x-1}$

hole: $(4, 3)$

$x - 4 = 0$

$x = 4$

$y = \dfrac{4+5}{4-1} = \dfrac{9}{3} = 3$

2) $f(x) = \dfrac{(x+2)(x+3)}{(x-5)(x+2)} = \dfrac{x+3}{x-5}$

$\underline{V.A.} : x = 1$

$f(x) = \dfrac{x+5}{x-1}$

$x - 1 = 0$

$x = 1$

hole: $\left(-2, -\frac{1}{7}\right)$ $\underline{V.A.} : x = 5$

$x + 2 = 0$ $x - 5 = 0$

$x = -2$ $x = 5$

$y = \dfrac{-2+3}{-2-5} = \dfrac{-1}{7}$

3) $f(x) = \dfrac{3x(x+1)}{(x+7)(x+1)}$

4) $f(x) = -\dfrac{4(x-3)}{(x-3)(x-1)}$

5) $f(x) = \dfrac{6(x-1)}{(x-1)(x+1)}$

6) $f(x) = \dfrac{x+6}{(x-2)(x+6)}$

7) $f(x) = \dfrac{3(x^2-9)}{x^2+x-6}$

8) $f(x) = \dfrac{x^2+4x+3}{x^2+2x-3}$

9) $f(x) = \dfrac{x^2 + 5x + 4}{x^2 + 4x}$

10) $f(x) = \dfrac{x^2 - 1}{x^2 - 3x - 4}$

11) $f(x) = \dfrac{3}{x - 4}$

12) $f(x) = \dfrac{x^2 + x - 2}{x^2 + 5x + 6}$

13) $f(x) = \dfrac{4}{x + 2}$

14) $f(x) = \dfrac{x^2 + 2x - 8}{4x^2 - 8x}$

$$\dfrac{(x-2)(x+4)}{4x(x-2)} = \dfrac{x+4}{4x}$$

No hole

$\underline{V.A.}$ $x = -2$

$x + 2 = 0$
$x = -2$

hole $(2, \frac{3}{4})$

$x - 2 = 0$
$x = 2$

$\dfrac{2+4}{4(2)} = \dfrac{6}{8}$

$\underline{V.A.}$
$4x = 0$
$x = 0$

15) $f(x) = \dfrac{x + 4}{x^2 + x - 12}$

16) $f(x) = \dfrac{x^2 + 4x + 3}{x^2 - 1}$