Finding Horizontal Asymptotes of rational functions

Degrees → highest exponent

Degree of numerator < Degree of denominator

(less than)

Ex. $f(x) = \frac{x}{x^2-1}$ degree 1 < degree 2

Degree of numerator = Degree of denominator

Ex. $f(x) = \frac{x^2+3x+2}{2x^2+4x}$ degree: 2, degree: 2

Degree of Numerator > Degree of Denominator

Ex. $f(x) = \frac{(3x^2+4x+1)}{x-2}$ degree: 2, degree: 1

Horizontal asymptote

is always $y = 0$

*Write as $y = 0$ not just 0

Example

$$f(x) = \frac{x^2 + 2x}{x^3 + x^2 - x + 1}$$

degree = 2

degree = 3

$2 < 3$

H.A.: $y = 0$

Make a fraction out of leading coefficients

*Looks like $y = \frac{\#}{\#}$

Example

$$f(x) = \frac{3x + 5}{4x - 1}$$

LC: 3

LC: 4

H.A.: $y = 3/4$

$$f(x) = \frac{1x^2 - 5x + 3}{3x^2 - 2x + 1}$$

LC: 1

LC: 3

H.A.: $y =$

No horizontal asymptote, but there can be a Slant asymptote

*use synthetic division to find slant asymptotes when numerator degree is exactly one larger than denom. degree

Example

$$f(x) = \frac{1x^2 - 1x + 1}{x - 5}$$

deg: 2

deg: 1

S.A.: $y = x + 4$

5	1	-1	1
	↓	5	20
	1	4	21

$$f(x) = \frac{x^2 - 3x - 10}{x - 1}$$