Graphing Rational Functions

Steps:

1. Find holes, asymptotes and intercepts
2. Sketch asymptotes with a dashed line. VA: ↕ HA: ←--→
3. Plot x-intercepts (on x-axis), y-intercepts (on y-axis) and holes (open circle).
4. Type function into graphing calculator or make a table.
   y= $(numerator) \div (denominator)$
   Remember Parentheses
   Graph to see the graph
5. Sketch graph to match calculator, It will go through intercepts and holes and "hug" asymptotes (Don't Touch)
6. Domain: All Possible x-values
   ℝ except for (V.A., holes, S.A.)
   Range: All Real numbers
   All possible y-values
   ℝ except for (H.A., holes, SA)

Ex. $f(x) = \frac{x^2-2x-3}{x^2+x-12} = \frac{(x+1)(x-3)}{(x+4)(x-3)} = \frac{x+1}{x+4}$

$\frac{3+1}{3+4} = \frac{4}{7}$

Hole: (3, 4/7)

VA: $x = -4$

HA: $y = 1$

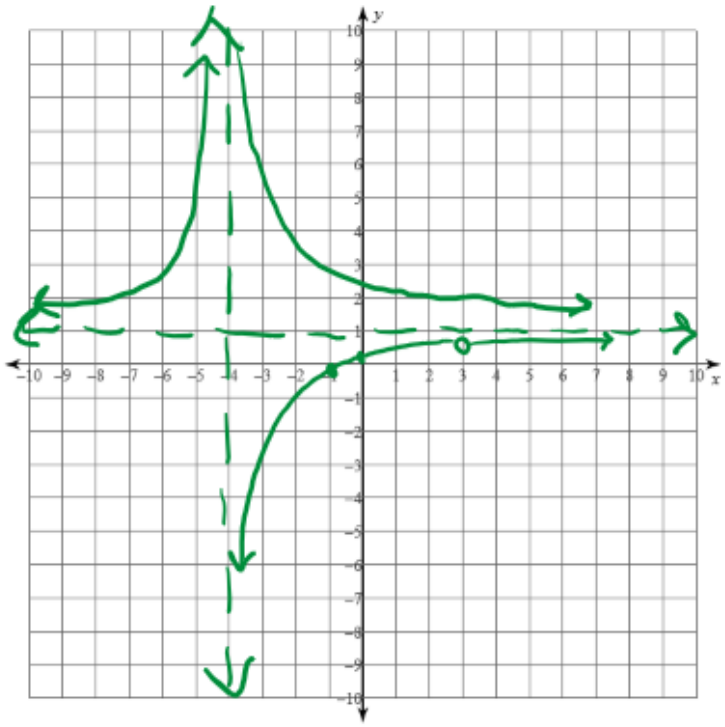
SA: None

x-int: (-1, 0)

y-int: $\frac{0+1}{0+4}$ = (0, 1/4)

Domain: ℝ except $x = -4, 3$

Range: ℝ except $y = 1, 4/7$



Ex. $f(x) = \frac{-3}{x^2-x-12} = \frac{-3}{(x-4)(x+3)}$ (y-int)

$y = \frac{-3}{(-4)^2-(4)-12} = \frac{-3}{8}$

Hole: None

VA: $x=4$ $x=-3$

HA: $y=0$

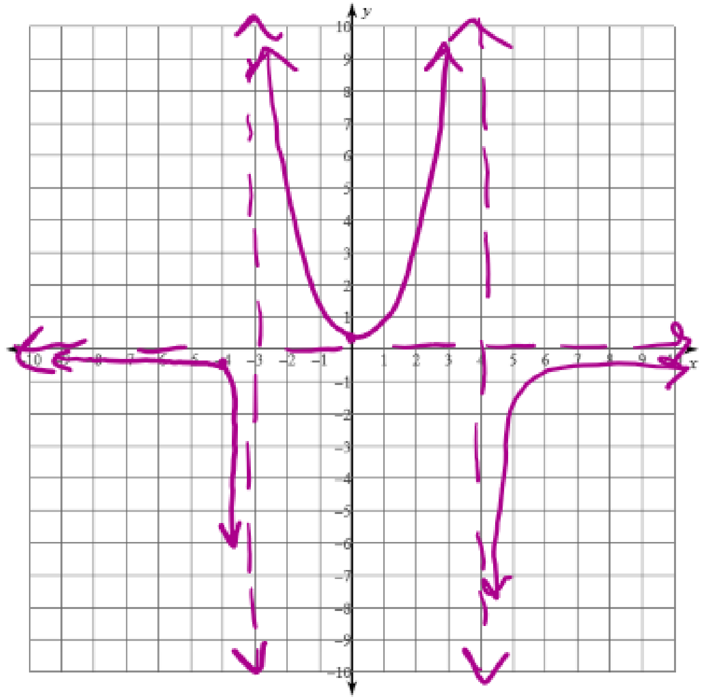
SA: None

x-int: None

y-int: $\frac{-3}{-12}$ (0, 1/4)

Domain: ℝ except $x=-3,4$

Range: ℝ except $y=0$



Ex. $f(x) = \frac{x^2+5x+6}{x+1} = \frac{(x+3)(x+2)}{x+1}$

Hole: None

VA: $x=-1$

HA: None

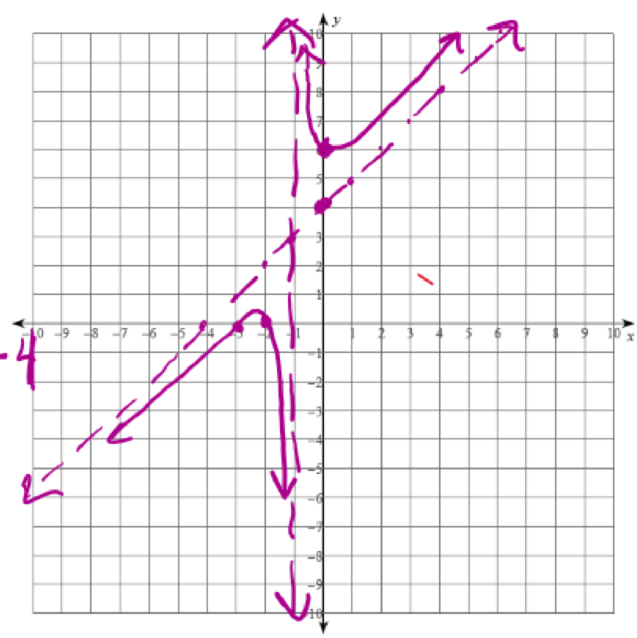
SA: $y=1x+4$ (synthetic division: −1 | 1 5 6 → 1 4 2)

x-int: (-3,0) (-2,0)

y-int: (0,6)

Domain: ℝ except $x=-1$, y-4

Range: ℝ except $y=x+4$



Domain for Slant

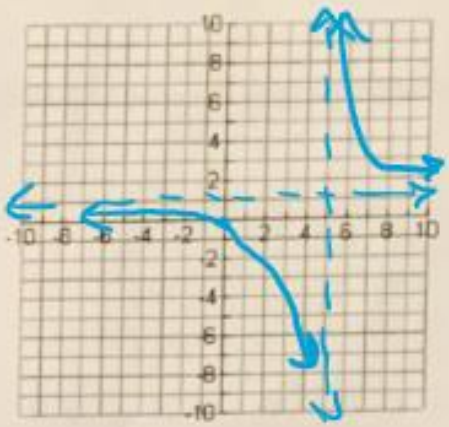
solve for x

$y = x+4$

$-4 \quad -4$

$y-4 = x$

7. $y = \frac{x}{x-5}$



Domain: $\mathbb{R}, x \neq 5$

Range: $\mathbb{R}, y \neq 1$

Vertical Asymptote: $x = 5$

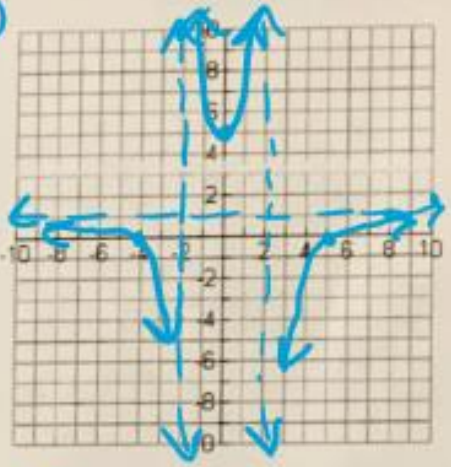
Horizontal Asymptote: $y = 1$

Slant Asymptote: None

Holes: None

x-int: (0,0) y-int: (0,0)

8. $y = \frac{x^2 - x - 20}{x^2 - 4} = \frac{(x-5)(x+4)}{(x-2)(x+2)}$



Domain: $\mathbb{R}, x \neq 2, x \neq -2$

Range: $\mathbb{R}, y \neq 1$

Vertical Asymptote: $x = 2, x = -2$

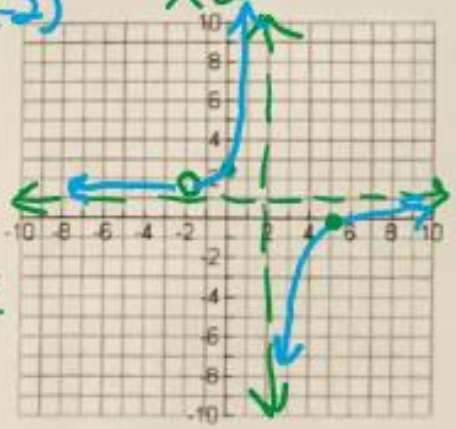
Horizontal Asymptote: $y = 1$

Slant Asymptote: None

Holes: None

x-int: (5,0) (-4,0) y-int: (0,5)

9. $y = \frac{x^2 - 3x - 10}{x^2 - 4} = \frac{(x-5)(x+2)}{(x+2)(x-2)} = \frac{x-5}{x-2}$



Domain: $\mathbb{R}\ x \neq 2, -2$

Range: $\mathbb{R}\ y \neq 1, 7/4$

Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 1$

Slant Asymptote: None

Holes: $(-2, 7/4)$

$\frac{-2-5}{-2-2} = \frac{-7}{-4}$

x-int: (5,0) y-int: $(0, 5/2)$