x^2

Solving Quadratic Equations by Factoring

How to Factor

Warm up: Recall multiplying polynomials…DISTRIBUTE TO MULTIPLY.

a) $2x(4x-3)$

$8x^2-6x$

b) $(x+5)(x+2)$

$x^2+2x+5x+10$

$x^2+7x+10$

> The process of factoring is the reverse of the process of distributing.
> The goal is to write an expression that is equivalent to the original, by dividing and "undistributing" any common factors.

FIRST: GREATEST COMMON FACTOR

For every factoring problem, you should begin by looking for a GCF.

Ex 1: Factor each expression.

a. $2x^2+8x$

$2x(x+4)$

b. $15x^2-35x$

$5x(3x-7)$

NEXT: After you have checked for a GCF, your strategy will depend on the number of terms in the polynomial.

THREE TERMS – SUM & PRODUCT STRATEGIES

When $a=1$:

If the trinomial (3 term) is a quadratic expression in standard form, $1x^2+bx+c$, AND $a=1$, find two factors of c which have a sum equal to b: then write the quadratic as the product of two binomial factors $(x+p)(x+q)$.

Ex 2: Factor each trinomial

a. $x^2+7x+12$

$(x+3)(x-$

S	P
7	12
~~13~~	~~1, 12~~
~~8~~	~~2, 6~~
7	$\boxed{3, 4}$

b. $x^2-10x+25$ (sum: -10; product: 25)

$(x-5)(x-5)$

$(x-5)^2$

S	P
-10	25
-10	-5, -5

c. $2x^2+4x-70$

$x^2+2x-35)$

$2(x-5)(x+7)$

S	P
2	-35
2	-5, 7

d. $5x^2-20x-225$

$5(x^2-4x-45)$

$5(x+5)(x-9)$

S	P
-4	-45
~~-44~~	~~1, -45~~
~~-12~~	~~3, -15~~
$\boxed{-4}$	$\boxed{5, -9}$

When $a \neq 1$: SLIDE AND DIVIDE

1) Multiply $a \cdot c$
2) Find two factors of $a \cdot c$ that have a sum equal to b
3) Set up two binomial factors: $(x+p)(x+q)$
4) Divide p and q by a…then simplify.

Ex 3: Factor each trinomial

a. $2x^2 - 9x - 18$

S	P
-9	-36
-9	3, -12

$(x+\frac{3}{2})(x-\frac{12}{2})$

$(2x+3)(x-6)$

b. $8x^2 - 30x + 7$

S	P
-30	56
-30	-2, -28

$(x-\frac{2}{8})(x-\frac{28}{8})$

$(x-\frac{1}{4})(x-\frac{7}{2})$

$(4x-1)(2x-7)$

c. $6x^2 - 5x - 4$

S	P
-5	-24
-5	3, -8

$(x+\frac{3}{6})(x-\frac{8}{6})$

$(x+\frac{1}{2})(x-\frac{4}{3})$

$(2x+1)(3x-4)$

d. $3x^2 - 20x + 32$

S	P
-20	96
	1, 96
	2, 48
	3, 32
	4, 24
	6, 16
✓	-8, -12

$(x-\frac{8}{3})(x-\frac{12}{3})$

$(3x-8)(x-4)$

TWO TERMS - DIFFERENCE OF SQUARES:

This is also a sum & product strategy, but notice that the value of the b-term in each example below is missing, therefore the sum of the factors must be zero.

Ex 4: Factor each binomial

a. $x^2 - 9$

S	P
0	-9
0	-3, 3

$(x-3)(x+3)$

b. $x^2 - 100$

S	P
0	-100
0	10, -10

$(x-10)(x+10)$

c. $x^2 - 81$

S	P
0	-81
0	-9, 9

$(x-9)(x+9)$

d. $x^2 - 4$

S	P
0	-4
0	-2, 2

$(x-2)(x+2)$

sub. perfect

What **pattern** do you notice about the factors of a **difference of squares**?

Ex 5: Use this pattern to factor the following

e. $25x^2 - 49$

$(5x-7)(5x+7)$

f. $100x^2 - 121$

$(10x-11)(10x+11)$

g. $16x^2 - 1$

$(4x-1)(4x+1)$

h. $x^2 + 25$

Not factorable

i. Using multiple strategies: $3x^2 - 75$

Solving Quadratic Equations by Factoring

According to the Zero Product Property, if the product of two quantities is equal to zero, then one of the quantities must equal zero.

Step 1: Arrange terms in standard form
Step 2: Factor
Step 3: Set each factor = 0
Step 4: Solve each mini-equation

Recall: Factoring Strategies

- Look for a GCF first!
- 2 terms: Difference of Squares?
- 3 terms: Sum & Product or Slide & Divide

Ex 6: Solve each equation by factoring.

a. $x^2 + 3x - 40 = 0$

S	P
3	-40
3	-5, 8

$(x-5)(x+8)=0$

$x-5=0 \quad x+8=0$

$x=5 \quad x=-8$

$\boxed{x=\{-8,5\}}$

b. $x^2 - 9x = 0$

$(x)(x-9)=0$

$x=0$ ✓ $x-9=0$

$x=9$

$x=\{0,9\}$

c. $x^2 - 3x - 28 = 0$

S	P
-3	-28
~~-27~~	~~1, -28~~
~~-12~~	~~2, -14~~
-3	$\boxed{4, -7}$

$(x+4)(x-7)=0$

$x+4=0 \quad x-7=0$

$x=-4 \quad x=7$

$x=\{-4,7\}$

d. $81x^2 - 100 = 0$ D.o.S.

$(9x+10)(9x-10)=0$

$9x+10=0 \quad 9x-10=0$

$-10 \quad -10$

$9x=-10$

$x=-\frac{10}{9} \quad x=10/9$

$x=\{-\frac{10}{9}, \frac{10}{9}\}$

e. $2x^2 - 24x = -72$

$+72 \quad +72$

$2x^2 - 24x + 72 = 0$

$2(x^2 - 12x + 36) = 0$

$x^2 - 12x + 36 = 0$

S	P
-12	36
~~1, 36~~	
~~2, 18~~	
~~3, 12~~	
~~4, 9~~	
-12	6, 6

$(x-6)(x-6) = 0$

$x - 6 = 0$

$\boxed{x = 6}$

g. $6x + 16 = x^2 + 9$

$-6x - 16 \quad -6x - 16$

$x^2 - 6x - 7 = 0$

S	P
-6	-7
-6	1, -7

$(x+1)(x-7) = 0$

$x + 1 = 0 \quad x - 7 = 0$

$x = -1 \quad x = 7$

$x = \{-1, 7\}$

i. $15x^2 - 10x = 0$

$(5x)(3x-2) = 0$

$\frac{5x}{5} = \frac{0}{5} \quad 3x - 2 = 0$

$x = 0 \quad 3x = 2$

$x = 2/3$

$x = \{0, 2/3\}$

a>1

f. $\boxed{3x^2 - 8x + 4 = 0}$

S	P
-8	12
-8	-2, -6

m by 3

$\frac{(x-2)}{3}\frac{(x-6)}{3} = 0$

$(3x-2)(x-2) = 0$

$3x - 2 = 0 \quad x - 2 = 0$

$3x = 2 \quad x = 2$

$x = 2/3$

$x = \{2/3, 2\}$

GCF 5

h. $5x^2 + 20x + 20 = 0$

$5(x^2 + 4x + 4) = 0$

$x^2 + 4x + 4 = 0$

S	P
4	4
4	2, 2

$(x+2)(x+2) = 0$

$x + 2 = 0$

$x = \{-2\}$

a>1 18

j. $18x^2 + 25x - 3 = 0$

S	P
25	-54
~~53~~	~~-1, 54~~
25	-2, 27

$\frac{(x-2)}{18}\frac{(x+27)}{18} = 0$

$(x - \frac{1}{9})(x + \frac{3}{2}) = 0$

$(9x-1)(2x+3) = 0$

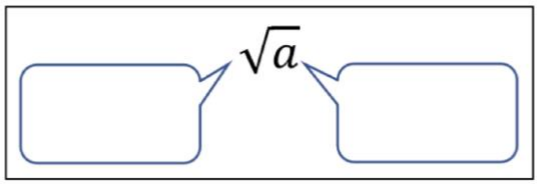
$9x - 1 = 0 \quad 2x + 3 = 0$

$9x = 1 \quad 2x = -3$

$x = 1/9 \quad x = -3/2$

$x = \{-3/2, 1/9\}$

Radical Expressions



Perfect Square: x^2	1	4	9	16	25	36	49	64	81	100	121
Square Roots: $\sqrt{x^2}$	1	2	3	4	5	6	7	8	9	10	11

Simplifying Radicals

A radical expression is simplified when there are…

1 no perfect square factors (other than 1) in the radicand

2 no fractions under the radical

3 no radicals in the denominator

To Simplify:

- Find the biggest perfect square factor of the radicand and evaluate its square root, bringing it outside the radical.
- The product of the remaining non-perfect-square factors will stay inside the radical.

Ex 1: Simplify each radical WITHOUT USING A CALCULATOR

a. $\sqrt{16} = 4$

b. $\sqrt{8}$ $\sqrt{4}\sqrt{2}$ $2\sqrt{2}$

c. $\sqrt{75}$ $\sqrt{25}\sqrt{3}$ $5\sqrt{3}$

d. $\sqrt{40}$ $\sqrt{2}\sqrt{10}$ $2\sqrt{10}$

e. $\sqrt{45}$ $\sqrt{9}\sqrt{5}$ $3\sqrt{5}$

f. $\sqrt{600}$ $\sqrt{100}\sqrt{6}$ $10\sqrt{6}$

When we simplify radicals, we are finding perfect squares – or PAIRS – of factors.

We will use a similar process to simplify radicals containing variables.

Ex 2: Simplify

a. $\sqrt{4x^2}$

$2x$

b. $\sqrt{98a^4b^{10}}$

$7a^2b^5\sqrt{2}$

c. $\sqrt{27z^3}$

$3z\sqrt{3z}$

d. $\sqrt{48xy^5z^9}$

$4y^2z^4\sqrt{3xyz}$

e. $\sqrt{100x^{12}y^7}$

$10x^6y^3\sqrt{y}$

f. $\sqrt{180a^3b^6}$

$6ab^3\sqrt{5a}$

Adding and Subtracting Radical Expressions

To simplify radical expressions involving addition and subtraction, we must combine "like radicals," which have identical radicands.

When adding and subtracting radicals, we will:

- Simplify each radical expression
- Combine the like radicals by adding or subtracting their coefficients, keeping the like radicand the same

Ex 4: Simplify

a. $\sqrt{3} + 5\sqrt{3}$

$6\sqrt{3}$

b. $2\sqrt{6} + \sqrt{24}$

$2\sqrt{6} + 2\sqrt{6}$

$4\sqrt{6}$

c. $3\sqrt{2} + \sqrt{5} - 4\sqrt{8}$

$3\sqrt{2} + \sqrt{5} - 4\,2\sqrt{2}$

$3\sqrt{2} + \sqrt{5} - 8\sqrt{2}$

$\boxed{-5\sqrt{2} + \sqrt{5}}$

d. $9\sqrt{40} - \sqrt{300} - \sqrt{90}$

$9\sqrt{4}\sqrt{10} - \sqrt{100}\sqrt{3} - \sqrt{9}\sqrt{10}$

$18\sqrt{10} - 10\sqrt{3} - 3\sqrt{10}$

$\boxed{15\sqrt{10} - 10\sqrt{3}}$

e. $\sqrt{72} - 4\sqrt{18}$

$\sqrt{36}\sqrt{2} - 4\sqrt{9}\sqrt{2}$

$6\sqrt{2} - 12\sqrt{2}$

$\boxed{-6\sqrt{2}}$

f. $\sqrt{20} + 2\sqrt{6} - \sqrt{80}$

$\sqrt{4}\sqrt{5} + 2\sqrt{6} - \sqrt{16}\sqrt{5}$

$2\sqrt{5} + 2\sqrt{6} - 4\sqrt{5}$

$\boxed{-2\sqrt{5} + 2\sqrt{6}}$

Multiplying Radical Expressions

When multiplying two radicals, we will multiply <u>OUTSIDE • OUTSIDE</u> and <u>INSIDE • INSIDE</u>, then simplify.

Ex 5: Simplify

a. $\sqrt{2} \cdot 5\sqrt{6}$

$5\sqrt{12}$

$2\sqrt{6}$

b. $3\sqrt{2} \cdot 5\sqrt{10}$

$15\sqrt{20}$

$30\sqrt{5}$

c. $\sqrt{3}(2 - \sqrt{3})$

$2\sqrt{3} - 9$

d. $2\sqrt{6} \cdot \sqrt{48}$

$2\sqrt{6} \cdot 4\sqrt{3}$

$8\sqrt{18}$

$24\sqrt{3}$

e. $(5+\sqrt{6})(2-\sqrt{2})$

$10-5\sqrt{2}+2\sqrt{6}-\sqrt{12}$

$10-5\sqrt{2}+2\sqrt{6}-2\sqrt{3}$

f. $(-4+\sqrt{6})(-1-\sqrt{6})$

$4+4\sqrt{6}-\sqrt{6}-6$

$-2+3\sqrt{6}$

g. $(2-\sqrt{3})(2+\sqrt{3})$

$4+3=7$

h. $(10+\sqrt{2})(10-\sqrt{2})$

$100-4$

96

Dividing with Radical Expressions & Rationalizing the Denominator

Simplify OUTSIDE/OUTSIDE and INSIDE/INSIDE, then rationalize the denominator to eliminate radicals from the bottom of the fraction, as needed. Simplify again, if necessary.

Ex 6: Simplify

a. $\frac{\sqrt{10}}{\sqrt{5}} = \frac{\sqrt{50}}{5} = \frac{5\sqrt{2}}{5} = \sqrt{2}$

b. $\frac{2\sqrt{15}}{\sqrt{3}} = \frac{2\sqrt{45}}{3} = \frac{6\sqrt{5}}{3} = 2\sqrt{5}$

c. $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

d. $\frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

e. $\frac{6\sqrt{2}}{2\sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$

f. $-\frac{9}{\sqrt{3}} = -3\sqrt{3}$

Algebra 2 Preview

Name_____

Factoring: GCF and a=1

Date_____ Period____

Factor the common factor out of each expression.

1) $12x^3 - 20x^2 + 12x$

$4x(3x^2 - 5x + 3)$

2) $40n^8 - 20n^4 + 5n^3$

$5n^3(8n^5 - 4n + 1)$

3) $24x^5 + 24x - 32$

$8(3x^5 + 3x - 4)$

4) $32v^6 - 72v + 8$

$8(4v^6 - 9v + 1)$

5) $-54 + 45n - 72n^2$

$9(-6 + 5n - 8n^2)$

6) $-21b + 70$

$7(-3b + 10)$

Factor each completely.

7) $x^2 + 6x + 8$

$(x+4)(x+2)$

8) $r^2 + 5r$

$r(r+5)$

9) $n^2 - 8n + 15$

$(n-5)(n-3)$

10) $n^2 + 5n - 36$

$(n+9)(n-4)$

11) $k^2 + k - 42$

$(k+7)(k-6)$

12) $3r^2 + 30r$

$3r(r+10)$

13) $b^2 - 5b + 6$

$(b-3)(b-2)$

14) $n^2 - 3n - 28$

$(n-7)(n+4)$

15) $2n^2 + 12n + 16$

$2(n^2 + 6n + 8)$

$2(n+4)(n+2)$

16) $6n^2 + 42n + 60$

$6(n^2 + 7n + 10)$

$6(n+5)(n+2)$

Algebra 2 Preview

Name_____

Factoring a > 1

Date_____ Period____

Factor each completely.

1) $3x^2 + 11x + 6$

$(3x+2)(x+3)$

2) $7x^2 + 12x - 4$

$(7x-2)(x+2)$

3) $5b^2 + 11b - 12$

$(5b-4)(b+3)$

4) $7v^2 + 52v - 32$

$(7v+4)(v-8)$

5) $6x^2 + 8x - 40$

$2(3x^2+4x-20)$

$2(3x+10)(x-2)$

6) $6a^2 - 39a - 72$

$3(2a^2-13a-24)$

$3(2a+3)(a-8)$

7) $10n^2 + 51n + 27$

$(5n+3)(2n+9)$

8) $9x^2 - 67x + 28$

$(9x-4)(x-7)$

9) $9x^2 - 64x + 60$

$(9x-10)(x-6)$

10) $6r^2 - 11r - 30$

$(3r-10)(2r+3)$

11) $54x^2 - 18x - 336$

$6(9x^2+3x-56)$

$6(3x-7)(3x+8)$

12) $36b^2 - 180b + 224$

$4(9b^2-45b+56)$

$4(3b-7)(3b-8)$

Algebra 2 Preview

Name_____

Difference Of Squares

Date_____ Period____

Factor each completely.

1) $4v^2 - 9$

$(2v+3)(2v-3)$

2) $16p^2 - 25$

$(4p^2-5)(4p+5)$

3) $4a^2 - 25$

$(2a-5)(2a+5)$

4) $a^2 + 9$

Not factorable

5) $16x^2 + 25$

Not factorable

6) $16n^2 - 9$

$(4n+3)(4n-3)$

7) $12p^2 - 3$

$3(4p^2-1)$

$3(2p+1)(2p-1)$

8) $16v^2 - 100$

$(4v+10)(4v-10)$

9) $50n^2 - 18$

$2(25n^2-9)$

$2(5n+9)(5n-9)$

10) $4a^2 - 1$

$(2a+1)(2a-1)$

11) $k^2 + 9$

$(k+3)(k-3)$

12) $5x^2 - 20$

$5(x^2-4)$

$5(x+2)(x-2)$

13) $18x^2 - 50$

$2(9x^2-25)$

$2(3x+5)(3x-5)$

14) $20k^2 + 125$

$5(4k^2+25)$

Algebra 2 Preview

Name_____

Factoring to Solve

Date_____ Period____

Solve each equation by factoring.

1) $m^2 - m - 12 = 0$

$m = \{-3, 4\}$

2) $x^2 + 10x + 16 = 0$

$x = \{-8, -2\}$

3) $x^2 + 6x + 8 = 0$

$x = \{-4, -2\}$

4) $x^2 + x - 30 = 0$

$x = \{-6, 5\}$

5) $n^2 - 9n + 18 = 0$

$n = \{3, 6\}$

6) $x^2 - 12x + 32 = 0$

$x = \{4, 8\}$

7) $7n^2 + 37n + 10 = 0$

$n = \{-5, -\frac{2}{7}\}$

8) $7n^2 - 41n - 6 = 0$

$n = \{-\frac{1}{7}, 6\}$

9) $5k^2 + 18k + 16 = 0$

$k = \{-2, -8/5\}$

10) $5n^2 + 41n + 8 = 0$

$n = \{-8, -\frac{1}{5}\}$

11) $56m^2 + 312m - 144 = 0$

$8(7m^2 + 39m - 18) = 0$

$m = \{-6, \frac{3}{7}\}$

12) $9a^2 - 78a + 144 = 0$

$3(3a^2 - 26a + 48) = 0$

$a = \{\frac{8}{3}, 6\}$

Algebra 2 Preview

Name_____

Simplifying Radicals

Date_____ Period____

Simplify.

1) $\sqrt{36} = 6$

2) $\sqrt{80} = 4\sqrt{5}$

3) $\sqrt{180} = 6\sqrt{5}$

4) $\sqrt{96} = 4\sqrt{6}$

5) $\sqrt{72} = 6\sqrt{2}$

6) $\sqrt{8} = 2\sqrt{2}$

7) $\sqrt{16} = 4$

8) $\sqrt{54} = 3\sqrt{6}$

9) $\sqrt{128x^4} = 8x^2\sqrt{2}$

10) $\sqrt{20x^3} = 2x\sqrt{5x}$

11) $\sqrt{243x^2} = 9x\sqrt{3}$

12) $\sqrt{20x^3y} = 2x\sqrt{5xy}$

13) $\sqrt{200xy^2} = 10y\sqrt{2x}$

14) $\sqrt{72x^4y^4} = 6x^2y^2\sqrt{2}$

15) $\sqrt{5184u^5v^2} = 72u^2v\sqrt{u}$

16) $\sqrt{4693x^{10}y^{12}} = 19x^5y^6\sqrt{13}$

Algebra 2 Preview

Name_____

Adding and Subtracting Radicals

Date_____ Period____

Simplify.

1) $2\sqrt{3}+2\sqrt{3}$

$4\sqrt{3}$

2) $2\sqrt{3}-\sqrt{3}$

$\sqrt{3}$

3) $-2\sqrt{6}-\sqrt{6}$

$-3\sqrt{6}$

4) $-2\sqrt{3}-3\sqrt{5}+3\sqrt{3}$

$\sqrt{3}-3\sqrt{5}$

5) $2\sqrt{2}-\sqrt{2}-\sqrt{2}$

0

6) $3\sqrt{6}-3\sqrt{6}-\sqrt{6}$

$-\sqrt{6}$

7) $-\sqrt{24}-2\sqrt{20}+3\sqrt{45}$

$-2\sqrt{6}-4\sqrt{5}+9\sqrt{5}$

$-2\sqrt{6}+5\sqrt{5}$

8) $3\sqrt{3}+2\sqrt{12}-\sqrt{12}$

$3\sqrt{3}+\sqrt{12}$

$3\sqrt{3}+2\sqrt{3}$

$5\sqrt{3}$

9) $2\sqrt{2}-2\sqrt{54}-\sqrt{2}$

$2\sqrt{2}-6\sqrt{6}-\sqrt{2}$

$\sqrt{2}-6\sqrt{6}$

10) $-\sqrt{5}+3\sqrt{20}-\sqrt{2}$

$-\sqrt{5}+6\sqrt{5}-\sqrt{2}$

$5\sqrt{5}-\sqrt{2}$

11) $-\sqrt{3}-3\sqrt{27}-3\sqrt{6}$

$-\sqrt{3}-9\sqrt{3}-3\sqrt{6}$

$-10\sqrt{3}-3\sqrt{6}$

12) $2\sqrt{8}+3\sqrt{5}-\sqrt{2}$

$4\sqrt{2}+3\sqrt{5}-\sqrt{2}$

$3\sqrt{2}+3\sqrt{5}$

13) $-\sqrt{6}+2\sqrt{6}-2\sqrt{2}-2\sqrt{6}$

$-\sqrt{6}-2\sqrt{2}$

14) $3\sqrt{6}-2\sqrt{5}-\sqrt{2}-2\sqrt{5}$

$3\sqrt{6}-4\sqrt{5}-\sqrt{2}$

15) $-2\sqrt{6}-\sqrt{2}-2\sqrt{6}-2\sqrt{2}$

$-4\sqrt{6}-3\sqrt{2}$

16) $-3\sqrt{6}-\sqrt{5}-3\sqrt{5}-2\sqrt{6}$

$-5\sqrt{6}-4\sqrt{5}$

Algebra 2 Preview Name_____

Multiply Radicals and Rationalizing Denominators Date_____ Period____

Simplify.

1) $\sqrt{15} \cdot \sqrt{12}$

$\sqrt{180}$

$\sqrt{36}\sqrt{5}$

$6\sqrt{5}$

2) $5\sqrt{5} \cdot -4\sqrt{5}$

$-20\sqrt{25}$

-100

3) $-4\sqrt{8} \cdot -3\sqrt{8}$

$12\sqrt{64}$

96

4) $\sqrt{6}(\sqrt{2} + \sqrt{6})$

$\sqrt{12} + \sqrt{36}$

$2\sqrt{3} + 6$

5) $\sqrt{2}(\sqrt{2} + 4)$

$\sqrt{4} + 4$

$2 + 4$

6

6) $4\sqrt{15}(3\sqrt{2} + 5\sqrt{3})$

$12\sqrt{30} + 20\sqrt{45}$

$12\sqrt{30} + 60\sqrt{5}$

7) $5\sqrt{6}(2\sqrt{3} - 5\sqrt{2})$

$10\sqrt{18} - 25\sqrt{12}$

$30\sqrt{2} - 50\sqrt{3}$

8) $(\sqrt{5} - 3)(\sqrt{5} + 3)$

$\sqrt{25} + \cancel{3\sqrt{5}} - \cancel{3\sqrt{5}} - 9$

$5 - 9$

-4

9) $(-3 + \sqrt{3})(-4 + \sqrt{3})$

$12 - 3\sqrt{3} - 4\sqrt{3} + \sqrt{9}$

$12 - 7\sqrt{3} + 3$

$15 - 7\sqrt{3}$

10) $(1 + 3\sqrt{5})(5 - 4\sqrt{5})$

$5 - 4\sqrt{5} + 15\sqrt{5} - 12\sqrt{25}$

$5 + 11\sqrt{5} - 60$

$-55 + 11\sqrt{5}$

11) $\frac{\sqrt{9}}{\sqrt{15}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

12) $\frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

13) $\frac{\sqrt{8}}{\sqrt{10}} = \frac{\sqrt{4}}{\sqrt{5}} = \frac{\sqrt{20}}{5}$

14) $-\frac{2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

15) $\frac{2\sqrt{4}}{4\sqrt{6}} = \frac{\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{6}$

16) $\frac{4\sqrt{3}}{\sqrt{15}} = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$

17) $\frac{4\sqrt{4}}{\sqrt{3}} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$

18) $\frac{5\sqrt{9}}{2\sqrt{15}} = \frac{5\sqrt{3}}{2\sqrt{5}} = \frac{5\sqrt{15}}{10} = \frac{\sqrt{15}}{2}$

19) $\frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}}$

20) $\frac{\sqrt{8}}{4\sqrt{6}} = \frac{\sqrt{4}}{4\sqrt{3}} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$